Exercise 5

Find the area of a parallelogram bounded by the y-axis, the line $x = 3$, the line $f(x) = 1 + 2x$, and the line parallel to $f(x)$ passing through $(2, 7)$.

Solution

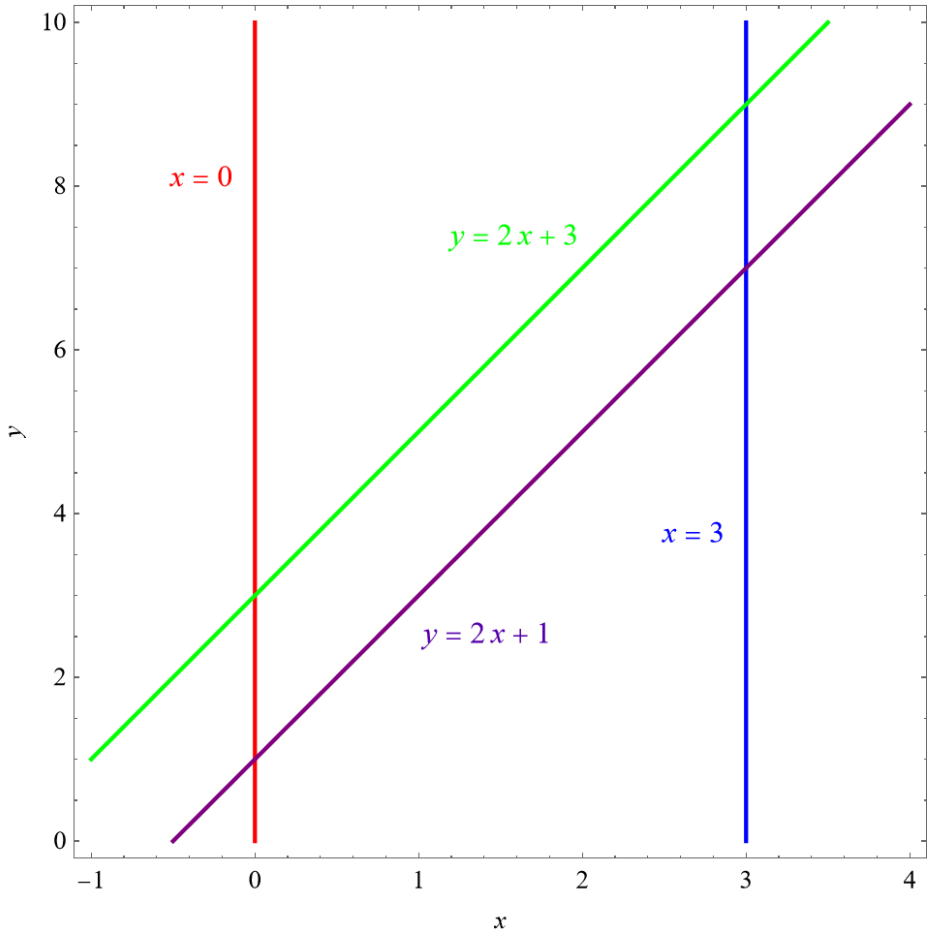
Start by writing equations of the lines that are given. The equation for the y-axis is $x = 0$, $x = 3$ is given, $y = 2x + 1$ is given, and the line parallel to $f(x)$ has the same slope (2) with an equation given by the point-slope formula.

$$y - 7 = 2(x - 2)$$

$$y - 7 = 2x - 4$$

$$y = 2x + 3$$

Now graph all of them.



The area enclosed within these lines is

$$A = \int_0^3 [(2x + 3) - (2x + 1)]\, dx = \int_0^3 (2x + 3 - 2x - 1)\, dx = \int_0^3 (2)\, dx = 2(3 - 0) = 6.$$